Užitečné vzorce:

$$\tan x = \frac{\sin x}{\cos x} = \frac{1}{\cot x}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2\cot x}$$

Úloha 1. V oboru reálných čísel řešte rovnici:

a) $\cos x = -\frac{1}{2}$

b) $\sin 2x = -\frac{\sqrt{2}}{2}$

c) $\tan\left(3x + \frac{\pi}{2}\right) = 1$

d) $\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$

e) $2\sin^2 x + \sin x - 1 = 0$

f) $2\sin^2 x - 3 = 3\cos x$

Úloha 2. Zjednodušte zadaný výraz a stanovte podmínky řešitelnosti

a) $\dfrac{\sin^2 x \cdot \cot x}{1 - \sin^2 x}$

b) $\dfrac{\tan x \cdot \tan 2x}{\tan x - \tan 2x}$

c) $\dfrac{\tan x + \cot x}{\tan x - \cot x}$

d) $\dfrac{1 - \cos 2x}{\sin 2x} + \dfrac{\sin 2x}{1 + \cos 2x}$

e) $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x}$

f) $\dfrac{1}{\cos^2 x} - \dfrac{\sin^3 x + \cos^3 x}{\sin x \cdot \cos^2 x + \cos^3 x}$

g) $\dfrac{1 - \cos^2 x}{1 + \tan^2 x} - \dfrac{\cos^2 x}{1 + \cot^2 x} + \dfrac{1}{\cos^2 x} - 1$

Vysledky

1.

a) $\bigcup_{k\in\mathbb{Z}} \left\{\frac{2}{3}\pi + 2k\pi; \frac{4}{3}\pi + 2k\pi\right\}$

b) $\bigcup_{k\in\mathbb{Z}} \left\{\frac{5}{8}\pi + k\pi; \frac{7}{8}\pi + k\pi\right\}$

c) $\bigcup_{k\in\mathbb{Z}} \left\{-\frac{1}{12}\pi + k\frac{\pi}{3}\right\}$

d) $\bigcup_{k\in\mathbb{Z}} \left\{\frac{7}{24}\pi + k\pi; \frac{11}{24}\pi + k\pi\right\}$

e) $\bigcup_{k\in\mathbb{Z}} \left\{\frac{1}{6}\pi + 2k\pi; \frac{5}{6}\pi + 2k\pi, \frac{3}{2}\pi + 2k\pi\right\}$

f) $\bigcup_{k\in\mathbb{Z}} \left\{\frac{4}{3}\pi + 2k\pi; \frac{5}{3}\pi + 2k\pi, (2k+1)\pi\right\}$

2.

a) $\tan x; \quad x \neq k \cdot \frac{\pi}{2}, k \in \mathbb{Z}$

b) $-\sin 2x; \quad x \neq (2k+1) \cdot \frac{\pi}{4}, x \neq (2k+1) \cdot \frac{\pi}{2}, k \in \mathbb{Z}$

c) $-\frac{1}{\cos 2x}; \quad x \neq k \cdot \frac{\pi}{2}, k \in \mathbb{Z}$

d) $2\tan x; \quad x \neq k \cdot \frac{\pi}{2}, k \in \mathbb{Z}$

e) $\frac{2}{\cos x}; \quad x \neq (2k+1) \cdot \frac{\pi}{2}, k \in \mathbb{Z}$

f) $\tan x; \quad x \neq (2k+1) \cdot \frac{\pi}{2}; x \neq -\frac{\pi}{4} + k\pi, k \in \mathbb{Z}$

g) $\tan^2 x; \quad x \neq k \cdot \frac{\pi}{2}, k \in \mathbb{Z}$