Soustavy obyčejných diferenciálních rovnic
Eulerova metoda
Řešené příklady

1.

$$\begin{aligned} x' &= 2x - y \\ y' &= 4x - 3y \end{aligned} \qquad \begin{aligned} x(0) &= 1 \\ y(0) &= -5 \end{aligned}$$

2.

$$\begin{aligned} x' &= -2x + y \\ y' &= -2x + y \end{aligned} \qquad \begin{aligned} x(0) &= 3 \\ y(0) &= 7 \end{aligned}$$

3.

$$\begin{aligned} x' &= x - 5y \\ y' &= x - y \end{aligned} \qquad \begin{aligned} x(0) &= 3 \\ y(0) &= 1 \end{aligned}$$

4.

$$\begin{aligned} x' &= 4x - y \\ y' &= 18x - 2y \end{aligned} \qquad \begin{aligned} x(0) &= 3 \\ y(0) &= 6 \end{aligned}$$

5.

$$\begin{aligned} x' &= -x - y \\ y' &= 9x + 5y \end{aligned} \qquad \begin{aligned} x(0) &= 2 \\ y(0) &= -1 \end{aligned}$$

6.

$$\begin{aligned} x' &= 3x - y \\ y' &= 4 - y \end{aligned} \qquad \begin{aligned} x(0) &= 3 \\ y(0) &= 1 \end{aligned}$$

7.

$$\begin{aligned} x' &= 4x - 5y + z \\ y' &= x - z \\ z' &= y - z \end{aligned}$$

8.

$$\begin{aligned} x' &= x + y + z \\ y' &= 2y + z \\ z' &= z \end{aligned}$$

9.

$$\begin{aligned} x' &= 2x \\ y' &= y - z \\ z' &= 2y - z \end{aligned}$$

(1) $x' = 2x - y \qquad x(0) = 1$

$y' = 4x - 3y \qquad y(0) = -5$

tj. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & -1 \\ 4 & -3 \end{pmatrix}}_{A} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$

$\det(A - r \cdot E) = 0 \quad : \quad \begin{vmatrix} 2-r & -1 \\ 4 & -3-r \end{vmatrix} = 0$

$(2-r)(-3-r) + 4 = 0$

$r^2 + r - 2 = 0$

$(r-1)(r+2) = 0 \quad \Rightarrow \quad r_1 = 1, \ r_2 = -2$ — vlastní čísla

vlastní vektory: $(A - r \cdot E) \cdot \bar{v} = \bar{o}$

$r_1 = 1 : \quad \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 4 & -4 & 0 \end{array}\right) \cdot (-4) \sim \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad v_1 - v_2 = 0$

$v_1 = 1 \Rightarrow v_2 = 1 \qquad \bar{v} = (1,1)$

1. řešení $\begin{pmatrix} x \\ y \end{pmatrix} = e^{t} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$r_2 = -2 : \quad \left(\begin{array}{cc|c} 4 & -1 & 0 \\ 4 & -1 & 0 \end{array}\right) \cdot (-1) \sim \left(\begin{array}{cc|c} 4 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad 4v_1 - v_2 = 0$

$v_1 = 1 \Rightarrow v_2 = 4 \qquad \bar{v} = (1,4)$

2. řešení $\begin{pmatrix} x \\ y \end{pmatrix} = e^{-2t} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

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obecní řešení $\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \cdot e^{t} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \cdot e^{-2t} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix} \qquad c_{1,2} \in \mathbb{R}$

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tj. $x = c_1 e^{t} + c_2 e^{-2t}$

$y = c_1 e^{t} + 4c_2 e^{-2t}$

$x(0) = 1 \quad : \quad 1 = c_1 + c_2$

$y(0) = -5 \quad : \quad -5 = c_1 + 4c_2 \qquad \Rightarrow \qquad c_1 = 3, \quad c_2 = -2$

$x = 3e^{t} - 2e^{-2t}$

$y = 3e^{t} - 8e^{-2t}$

② $x' = -2x + y \qquad x(0) = 3$
$y' = -2x + y \qquad y(0) = 7$

$$\begin{vmatrix} -2-r & 1 \\ -2 & 1-r \end{vmatrix} = 0 \qquad (-2-r)(1-r) + 2 = 0$$
$$r^2 + r = 0$$
$$r(r+1) = 0 \quad \Rightarrow r_1 = 0\,,\ r_2 = -1$$

$r_1 = 0$: $\left(\begin{array}{cc|c} -2 & 1 & 0 \\ -2 & 1 & 0 \end{array}\right) \cdot(-1) \sim \left(\begin{array}{cc|c} -2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\quad -2v_1 + v_2 = 0$, $v_1 = 1 \Rightarrow v_2 = 2$ $\quad \bar{v} = (1,2)$

1. řešení $\begin{pmatrix} x \\ y \end{pmatrix} = e^{0t} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$r_2 = -1$: $\left(\begin{array}{cc|c} -1 & 1 & 0 \\ -2 & 2 & 0 \end{array}\right) \cdot(-2) \sim \left(\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\quad -v_1 + v_2 = 0$, $v_1 = 1 \Rightarrow v_2 = 1$ $\quad \bar{v} = (1,1)$

2. řešení $\begin{pmatrix} x \\ y \end{pmatrix} = e^{-t} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \cdot e^{-t} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad c_{1,2} \in \mathbb{R}$$

$x = c_1 + c_2 e^{-t}$
$y = 2c_1 + c_2 e^{-t}$

$x(0) = 3$: $3 = c_1 + c_2$
$y(0) = 7$: $7 = 2c_1 + c_2$
$\Rightarrow c_1 = 4$, $c_2 = -1$

$x = 4 - e^{-t}$
$y = 8 - e^{-t}$

③ $x' = x - 5y \qquad x(0) = 3$

$y' = x - y \qquad y(0) = +1$

$$\begin{vmatrix} 1-r & -5 \\ 1 & -1-r \end{vmatrix} = 0 \qquad (1-r)(-1-r)+5=0$$

$$r^2+4=0 \quad \Rightarrow \quad r_{1,2} = \pm 2i$$

$$r_1 = 2i: \left(\begin{array}{cc|c} 1-2i & -5 & 0 \\ 1 & -1-2i & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1-2i & 0 \\ 1-2i & -5 & 0 \end{array}\right) \cdot(1+2i) \sim \left(\begin{array}{cc|c} 1 & -1-2i & 0 \\ 5 & -5-10i & 0 \end{array}\right) \cdot(-5) \sim$$

$$\sim \left(\begin{array}{cc|c} 1 & -1-2i & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad \begin{array}{l} v_1 + (-1-2i)v_2 = 0 \\ v_2 = 1 \Rightarrow v_1 = 1+2i \end{array} \qquad \bar{v} = (1+2i, 1)$$

$$\bar{v} \cdot e^{r_1 t} = \begin{pmatrix} 1+2i \\ 1 \end{pmatrix} \cdot e^{2it} = \begin{pmatrix} 1+2i \\ 1 \end{pmatrix} \cdot (\cos 2t + i \cdot \sin 2t) =$$

$$= \begin{pmatrix} \cos 2t + i \cdot \sin 2t + 2i \cdot \cos 2t - 2 \cdot \sin 2t \\ \cos 2t + i \cdot \sin 2t \end{pmatrix} = \qquad \left| \begin{array}{l} \text{pozn.} \\ i \cdot i = -1 \end{array} \right|$$

$$= \underbrace{\begin{pmatrix} \cos 2t - 2\sin 2t \\ \cos 2t \end{pmatrix}}_{1.\ \text{řešení}} + i \cdot \underbrace{\begin{pmatrix} 2\cos 2t + \sin 2t \\ \sin 2t \end{pmatrix}}_{2.\ \text{řešení}}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \cdot \begin{pmatrix} \cos 2t - 2\sin 2t \\ \cos 2t \end{pmatrix} + c_2 \cdot \begin{pmatrix} 2\cos 2t + \sin 2t \\ \sin 2t \end{pmatrix} \qquad c_{1,2} \in \mathbb{R}$$

$$x = (c_1 + 2c_2) \cdot \cos 2t + (-2c_1 + c_2) \cdot \sin 2t$$

$$y = c_1 \cdot \cos 2t + c_2 \cdot \sin 2t$$

$$\begin{array}{ll} x(0) = 3: & 3 = c_1 + 2c_2 \\ y(0) = +1: & +1 = c_1 \end{array} \quad \Rightarrow \quad \begin{array}{l} c_1 = 1 \\ c_2 = 1 \end{array}$$

$$x = 3\cos 2t - \sin 2t$$

$$y = \cos 2t + \sin 2t$$

(4) $x' = 4x - y \qquad x(0) = 3$
$y' = 18x - 2y \qquad y(0) = 6$

$$\begin{vmatrix} 4-r & -1 \\ 18 & -2-r \end{vmatrix} = 0 \qquad (4-r)(-2-r) + 18 = 0$$

$$r^2 - 2r + 10 = 0 \Rightarrow r_{1,2} = \frac{2 \pm \sqrt{-36}}{2} = 1 \pm 3i$$

$$r_1 = 1+3i: \quad \left(\begin{array}{cc|c} 3-3i & -1 & 0 \\ 18 & -3-3i & 0 \end{array}\right) \cdot (3+3i) \sim \left(\begin{array}{cc|c} 18 & -3-3i & 0 \\ 18 & -3-3i & 0 \end{array}\right) \cdot(-1) \sim \left(\begin{array}{cc|c} 18 & -3-3i & 0 \\ 0 & 0 & 0 \end{array}\right) :(3+3i) \sim$$

$$\sim \left(\begin{array}{cc|c} 3-3i & -1 & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad (3-3i)v_1 - v_2 = 0 \qquad \bar{v} = (1, 3-3i)$$

$$v_1 = 1 \Rightarrow v_2 = 3-3i$$

$$\bar{v} \cdot e^{r_1 t} = \begin{pmatrix} 1 \\ 3-3i \end{pmatrix} \cdot e^{(1+3i)t} = \begin{pmatrix} 1 \\ 3-3i \end{pmatrix} \cdot \left(e^t \cdot \cos 3t + i \cdot e^t \sin 3t\right) =$$

$$= \begin{pmatrix} e^t \cos 3t + i \cdot e^t \sin 3t \\ 3e^t \cos 3t + 3ie^t \sin 3t - 3ie^t \cos 3t + 3e^t \sin 3t \end{pmatrix} =$$

$$= \underbrace{\begin{pmatrix} e^t \cos 3t \\ 3e^t \cos 3t + 3e^t \sin 3t \end{pmatrix}}_{1.\ \text{řešení}} + i \cdot \underbrace{\begin{pmatrix} e^t \sin 3t \\ -3e^t \cos 3t + 3e^t \sin 3t \end{pmatrix}}_{2.\ \text{řešení}}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \cdot \begin{pmatrix} e^t \cos 3t \\ 3e^t \cos 3t + 3e^t \sin 3t \end{pmatrix} + c_2 \cdot \begin{pmatrix} e^t \sin 3t \\ -3e^t \cos 3t + 3e^t \sin 3t \end{pmatrix} \qquad c_{1,2} \in \mathbb{R}$$

$$x = c_1 e^t \cos 3t + c_2 e^t \sin 3t$$
$$y = (3c_1 - 3c_2) e^t \cos 3t + (3c_1 + 3c_2) e^t \sin 3t$$

$$\begin{aligned} x(0) = 3 &: \quad 3 = c_1 \\ y(0) = 6 &: \quad 6 = 3c_1 - 3c_2 \end{aligned} \Rightarrow \begin{aligned} c_1 &= 3 \\ c_2 &= 1 \end{aligned}$$

$$x = 3e^t \cos 3t + e^t \sin 3t$$
$$y = 6e^t \cos 3t + 12e^t \sin 3t$$

(5) $x' = -x - y \qquad x(0) = 2$

$y' = 9x + 5y \qquad y(0) = -1$

$$\begin{vmatrix} -1-r & -1 \\ 9 & 5-r \end{vmatrix} = 0 \qquad (-1-r)(5-r) + 9 = 0$$

$$r^2 - 4r + 4 = 0$$

$$(r-2)^2 = 0 \quad \Rightarrow \quad r_1 = r_2 = 2$$

$r_1 = 2$: $\left(\begin{array}{cc|c} -3 & -1 & 0 \\ 9 & 3 & 0 \end{array}\right) \cdot 3 \sim \left(\begin{array}{cc|c} -3 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\quad -3v_1 - v_2 = 0 \quad v_1 = 1 \Rightarrow v_2 = -3 \quad \bar{v} = (1, -3)$ vlastní vektor

1\. řešení $\begin{pmatrix} x \\ y \end{pmatrix} = e^{2t} \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix} \quad \left(= e^{2t} \cdot \bar{v} \right)$

zobecněný vlastní vektor : $(A - r_1 \cdot E) \cdot \bar{w} = \bar{v}$

$\left(\begin{array}{cc|c} -3 & -1 & 1 \\ 9 & 3 & -3 \end{array}\right) \cdot 3 \sim \left(\begin{array}{cc|c} -3 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right)$ $\quad -3w_1 - w_2 = 1 \quad w_1 = 1 \Rightarrow w_2 = -4 \quad \bar{w} = (1, -4)$

2\. řešení $\begin{pmatrix} x \\ y \end{pmatrix} = e^{2t} \cdot \left(\bar{w} + t \cdot \bar{v} \right) = e^{2t} \cdot \left(\begin{pmatrix} 1 \\ -4 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right)$

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obecné řešení $\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \cdot e^{2t} \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix} + c_2 \cdot e^{2t} \cdot \left(\begin{pmatrix} 1 \\ -4 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right) \qquad c_{1,2} \in \mathbb{R}$

$$x = (c_1 + c_2) e^{2t} + c_2 t e^{2t}$$

$$y = (-3c_1 - 4c_2) e^{2t} - 3c_2 t e^{2t}$$

$x(0) = 2 \quad : \quad 2 = c_1 + c_2$

$y(0) = -1 \quad : \quad -1 = -3c_1 - 4c_2$

$\Rightarrow \quad c_1 = 7, \quad c_2 = -5$

$$x = 2e^{2t} - 5te^{2t}$$

$$y = -e^{2t} + 15te^{2t}$$

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$$x' = 3x - y \qquad x(0) = 3$$
$$y' = 4x - y \qquad y(0) = 1$$

$$\begin{vmatrix} 3-r & -1 \\ 4 & -1-r \end{vmatrix} = 0 \qquad (3-r)(-1-r) + 4 = 0$$
$$r^2 - 2r + 1 = 0$$
$$(r-1)^2 = 0 \qquad r_1 = r_2 = 1$$

$$r_1 = 1 : \left(\begin{array}{cc|c} 2 & -1 & 0 \\ 4 & -2 & 0 \end{array}\right) \cdot(-2) \sim \left(\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad \begin{array}{l} 2v_1 - v_2 = 0 \\ v_1 = 1 \Rightarrow v_2 = 2 \end{array} \qquad \bar{v} = (1, 2)$$

1. řešení $\begin{pmatrix} x \\ y \end{pmatrix} = e^t \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$(A - r_1 E)\cdot \bar{w} = \bar{v} : \left(\begin{array}{cc|c} 2 & -1 & 1 \\ 4 & -2 & 2 \end{array}\right) \cdot(-2) \sim \left(\begin{array}{cc|c} 2 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right) \qquad \begin{array}{l} 2w_1 - w_2 = 1 \\ w_1 = 1 \Rightarrow w_2 = 1 \\ \bar{w} = (1,1) \end{array}$$

2. řešení $\begin{pmatrix} x \\ y \end{pmatrix} = e^t \cdot \left(\overset{\bar{w}}{\begin{pmatrix} 1 \\ 1 \end{pmatrix}} + t \cdot \overset{\bar{v}}{\begin{pmatrix} 1 \\ 2 \end{pmatrix}} \right)$

$$\begin{pmatrix} x \\ y \end{pmatrix} = c_1 e^t \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^t \cdot \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right) \qquad c_{1,2} \in \mathbb{R}$$

$$x = (c_1 + c_2) \cdot e^t + c_2 t e^t$$
$$y = (2c_1 + c_2) e^t + 2c_2 t e^t$$

$$\begin{array}{lll} x(0) = 3 : & 3 = c_1 + c_2 & \\ y(0) = 1 : & 1 = 2c_1 + c_2 & \end{array} \Rightarrow \begin{array}{l} c_1 = -2 \\ c_2 = 5 \end{array}$$

$$x = 3e^t + 5te^t$$
$$y = e^t + 10te^t$$

7)
$$x' = 4x - 5y + z$$
$$y' = x - z$$
$$z' = y - z$$

$$\begin{vmatrix} 4-r & -5 & 1 \\ 1 & -r & -1 \\ 0 & 1 & -1-r \end{vmatrix} = 0$$

$$(4-r)(-r)(-1-r) + 1 + 4 - r + 5(-1-r) = 0$$
$$r^3 - 3r^2 + 2r = 0$$
$$r(r-1)(r-2) = 0$$
$r_1 = 0$, $r_2 = 1$, $r_3 = 2$

$$r_1 = 0: \left(\begin{array}{ccc|c} 4 & -5 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 4 & -5 & 1 & 0 \end{array}\right)^{(-4)} \sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -5 & 5 & 0 \end{array}\right)^{\cdot 5} \sim \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$v_1 - v_3 = 0$
$v_2 - v_3 = 0$

$v_3 = 1 \Rightarrow v_2 = 1,\ v_1 = 1$
$\bar{v} = (1,1,1)$

1\. řešení: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = e^{0t} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$r_2 = 1: \left(\begin{array}{ccc|c} 3 & -5 & 1 & 0 \\ 1 & -1 & -1 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 3 & -5 & 1 & 0 \end{array}\right)^{(-3)} \sim \left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 4 & 0 \end{array}\right)^{2} \sim \left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$v_1 - v_2 - v_3 = 0$
$v_2 - 2v_3 = 0$

$v_3 = 1 \Rightarrow v_2 = 2,\ v_1 = 3$
$\bar{v} = (3,2,1)$

2\. řešení $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = e^{t} \cdot \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

$$r_3 = 2: \left(\begin{array}{ccc|c} 2 & -5 & 1 & 0 \\ 1 & -2 & -1 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 1 & -3 & 0 \\ 2 & -5 & 1 & 0 \end{array}\right)^{(-2)} \sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & -1 & 3 & 0 \end{array}\right)^{+} \sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$v_1 - 2v_2 - v_3 = 0$
$v_2 - 3v_3 = 0$

$v_3 = 1 \Rightarrow v_2 = 3,\ v_1 = 7$
$\bar{v} = (7,3,1)$

3\. řešení: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = e^{2t} \cdot \begin{pmatrix} 7 \\ 3 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 \cdot e^{t} \cdot \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + c_3 \cdot e^{2t} \cdot \begin{pmatrix} 7 \\ 3 \\ 1 \end{pmatrix} \qquad c_{1,2,3} \in \mathbb{R}$$

$$x = c_1 + 3c_2 e^{t} + 7c_3 e^{2t}$$
$$y = c_1 + 2c_2 e^{t} + 3c_3 e^{2t}$$
$$z = c_1 + c_2 e^{t} + c_3 e^{t}$$

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$$x' = x + y + z$$
$$y' = 2y + z$$
$$z' = z$$

$$\begin{vmatrix} 1-r & 1 & 1 \\ 0 & 2-r & 1 \\ 0 & 0 & 1-r \end{vmatrix} = 0 \qquad (1-r)(2-r)(1-r) = 0$$
$$r_1 = 2, \quad r_2 = r_3 = 1$$

$$r_1 = 2: \quad \left(\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right) \begin{matrix} \\ \urcorner + \\ \lrcorner \end{matrix} \sim \left(\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$-v_1 + v_2 + v_3 = 0 \qquad v_2 = 1 \Rightarrow v_1 = 1$$
$$v_3 = 0$$
$$\vec{v} = (1, 1, 0)$$

1. řešení: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = e^{2t} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$$r_{2,3} = 1: \quad \left(\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \begin{matrix} \urcorner (-1) \\ \lrcorner \\ \\ \end{matrix} \sim \left(\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$v_2 + v_3 = 0 \qquad h(A - E) = 1$$
$\Rightarrow$ 2 LNZ vlastní v.

a) $v_1 = 1, v_2 = 0 \Rightarrow v_3 = 0 \quad (1, 0, 0)$

b) $v_1 = 0, v_2 = 1 \Rightarrow v_3 = -1 \quad (0, 1, -1)$

2. řešení: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = e^{t} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ 3. řešení: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = e^{t} \cdot \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 \cdot e^{2t} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 \cdot e^{t} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_3 \cdot e^{t} \cdot \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \qquad c_{1,2,3} \in \mathbb{R}$$

$$x = c_1 e^{2t} + c_2 e^{t}$$
$$y = c_1 e^{2t} + c_3 e^{t}$$
$$z = -c_3 e^{t}$$

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$$x' = 2x$$
$$y' = y - z$$
$$z' = 2y - z$$

$$\begin{vmatrix} 2-r & 0 & 0 \\ 0 & 1-r & -1 \\ 0 & 2 & -1-r \end{vmatrix} = 0 \qquad (2-r)(1-r)(-1-r) + 2(2-r) = 0$$
$$(2-r)(r^2+1) = 0 \qquad r_1 = 2, \quad r_{2,3} = \pm i$$

$$r_1 = 2: \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 2 & -3 & 0 \end{array}\right) \cdot 2 \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -5 & 0 \end{array}\right) \qquad \begin{array}{l} v_2 + v_3 = 0 \\ -5v_3 = 0 \end{array} \quad v_3 = 0,\ v_2 = 0$$

zvolíme $v_1 = 1$

$$\bar{v} = (1, 0, 0)$$

1. řešení: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = e^{2t} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$r_2 = i: \left(\begin{array}{ccc|c} 2-i & 0 & 0 & 0 \\ 0 & 1-i & -1 & 0 \\ 0 & 2 & -1-i & 0 \end{array}\right) \begin{array}{l} \cdot(2+i) \\ \cdot(1+i) \\ \end{array} \sim \left(\begin{array}{ccc|c} 5 & 0 & 0 & 0 \\ 0 & 2 & -1-i & 0 \\ 0 & 2 & -1-i & 0 \end{array}\right) \cdot(-1) \sim \left(\begin{array}{ccc|c} 5 & 0 & 0 & 0 \\ 0 & 2 & -1-i & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$5v_1 = 0 \Rightarrow v_1 = 0$$
$$2v_2 + (-1-i)v_3 = 0 \qquad v_3 = 2 \Rightarrow v_2 = 1+i \qquad \bar{v} = (0, 1+i, 2)$$

$$\bar{v} \cdot e^{r_2 t} = \begin{pmatrix} 0 \\ 1+i \\ 2 \end{pmatrix} \cdot e^{it} = \begin{pmatrix} 0 \\ 1+i \\ 2 \end{pmatrix} \cdot (\cos t + i \cdot \sin t) = \begin{pmatrix} 0 \\ \cos t + i \cdot \sin t + i \cdot \cos t - \sin t \\ 2\cos t + 2i\sin t \end{pmatrix} =$$

$$= \underbrace{\begin{pmatrix} 0 \\ \cos t - \sin t \\ 2\cos t \end{pmatrix}}_{2.\ \text{řešení}} + i \cdot \underbrace{\begin{pmatrix} 0 \\ \cos t + \sin t \\ 2\sin t \end{pmatrix}}_{3.\ \text{řešení}}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = c_1 \cdot e^{2t} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 \cdot \begin{pmatrix} 0 \\ \cos t - \sin t \\ 2\cos t \end{pmatrix} + c_3 \cdot \begin{pmatrix} 0 \\ \cos t + \sin t \\ 2\sin t \end{pmatrix} \qquad c_{1,2,3} \in \mathbb{R}$$

$$x = c_1 \cdot e^{2t}$$

$$y = (c_2 + c_3) \cdot \cos t + (-c_2 + c_3) \cdot \sin t$$

$$z = 2c_2 \cdot \cos t + 2c_3 \cdot \sin t$$
