

INTEGRACE – VZORCE

Tabulkové integrály

$$1) \int 0 \, dx = c$$

$$2) \int x^n \, dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$$

$$3) \int a^x \, dx = \frac{a^x}{\ln a} + c, \quad a > 0$$

$$4) \int e^x \, dx = e^x + c$$

$$5) \int \frac{1}{x} \, dx = \ln|x| + c$$

$$6) \int \cos x \, dx = \sin x + c$$

$$7) \int \sin x \, dx = -\cos x + c$$

$$8) \int \frac{1}{\cos^2 x} \, dx = \tan x + c$$

$$9) \int \frac{1}{\sin^2 x} \, dx = -\cot x + c$$

$$10) \int \frac{1}{x^2 + 1} \, dx = \arctan x + c$$

$$11) \int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$12) \int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + c$$

$$13) \int \frac{1}{\sqrt{a^2-x^2}} \, dx = \arcsin \frac{x}{a} + c$$

$$14) \int \frac{1}{x^2-1} \, dx = \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + c$$

$$15) \int \frac{1}{x^2-a^2} \, dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$$

$$16) \int \frac{1}{\sqrt{x^2+a}} \, dx = \ln \left| x + \sqrt{x^2+a} \right| + c$$

Další vztahy pro integraci

$$17) \int (f(x) + g(x)) \, dx = \int f(x) \, dx + \int g(x) \, dx \quad 18) \int k \cdot f(x) \, dx = k \int f(x) \, dx$$

$$19) \int f(ax+b) \, dx = \frac{1}{a} F(ax+b) + c$$

$$20) \int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + c$$

$$21) \int f'(x) \cdot f(x) \, dx = \frac{f^2(x)}{2} + c$$

Goniometrické vztahy

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

Metoda per partes

$$\int u(x) \cdot v'(x) \, dx = u(x) \cdot v(x) - \int u'(x) \cdot v(x) \, dx$$

Substituční metoda

$$\int f(g(x)) \cdot g'(x) \, dx = \left| \begin{array}{l} t = g(x) \\ dt = g'(x) \, dx \end{array} \right| = \int f(t) \, dt = F(t) + c = F(g(x)) + c$$

Substituce pro integraci goniometrických funkcí

$$t = \tan x : \quad dx = \frac{1}{1+t^2} dt; \quad \sin x = \frac{t}{\sqrt{1+t^2}}; \quad \cos x = \frac{1}{\sqrt{1+t^2}}$$

$$t = \tan \frac{x}{2} : \quad dx = \frac{2}{1+t^2} dt; \quad \sin x = \frac{2t}{1+t^2}; \quad \cos x = \frac{1-t^2}{1+t^2}$$