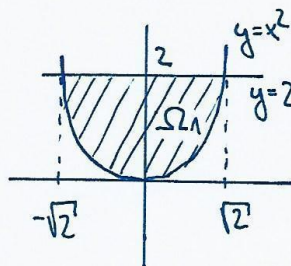
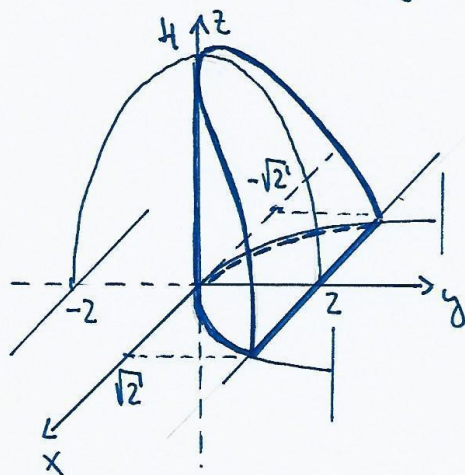


$$V(\Omega) = \iiint_{\Omega} dx dy dz$$

① $\Omega: y=x^2, z=4-y^2, z=0, V(\Omega)=?$

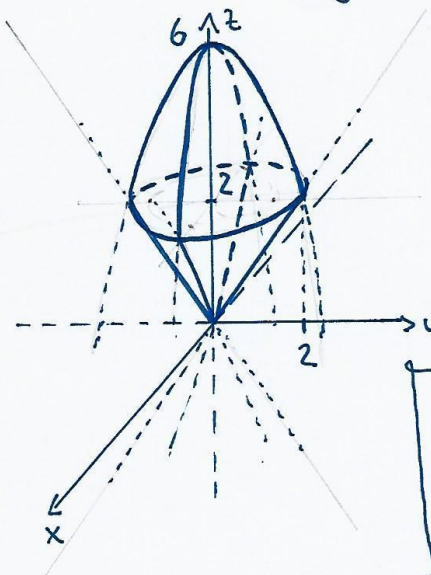


$$\begin{aligned} -\sqrt{2} &\leq x \leq \sqrt{2} \\ x^2 &\leq y \leq 2 \\ 0 &\leq z \leq 4-y^2 \end{aligned}$$

resp. $0 \leq x \leq \sqrt{2}$ (2x)

$$\begin{aligned} V(\Omega) &= \iiint_{\Omega} dx dy dz = 2 \cdot \int_0^{\sqrt{2}} dx \int_{x^2}^2 dy \int_0^{4-y^2} dz = 2 \cdot \int_0^{\sqrt{2}} dx \int_{x^2}^2 [z]_0^{4-y^2} dy = \\ &= 2 \cdot \int_0^{\sqrt{2}} dx \int_{x^2}^2 (4-y^2) dy = 2 \cdot \int_0^{\sqrt{2}} \left[4y - \frac{y^3}{3} \right]_{x^2}^2 dx = 2 \cdot \int_0^{\sqrt{2}} \left(8 - \frac{8}{3} - 4x^2 + \frac{x^6}{3} \right) dx = \\ &= 2 \cdot \int_0^{\sqrt{2}} \left(\frac{16}{3} - 4x^2 + \frac{x^6}{3} \right) dx = 2 \cdot \left[\frac{16x}{3} - \frac{4x^3}{3} + \frac{x^7}{21} \right]_0^{\sqrt{2}} = \\ &= 2 \cdot \left(\frac{16\sqrt{2}}{3} - \frac{8\sqrt{2}}{3} + \frac{8\sqrt{2}}{21} \right) = \underline{\underline{\frac{128\sqrt{2}}{3}}} \end{aligned}$$

② $\Omega : z^2 \geq x^2 + y^2, z \leq 6 - x^2 - y^2, V(\Omega) = ?$



$$\begin{aligned} x &= \rho \cdot \cos \varphi \\ y &= \rho \cdot \sin \varphi \\ z &= u \\ \lambda &= \rho \end{aligned}$$

$$\begin{aligned} z^2 &= x^2 + y^2 & z &= 6 - x^2 - y^2 \\ u^2 &= \rho^2 & u &= 6 - \rho^2 \\ u &= \rho \end{aligned}$$

$$\rho = 6 - \rho^2$$

$$\rho^2 + \rho - 6 = 0$$

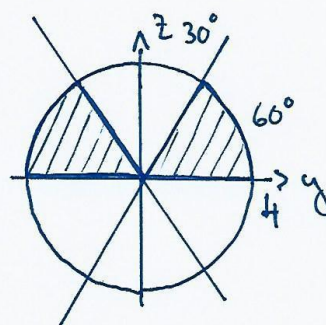
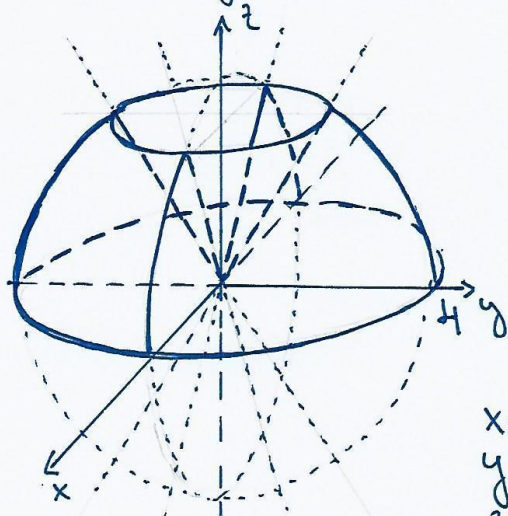
$$(\rho - 2)(\rho + 3) = 0$$

$$\rho = 2 \quad \rho = -3$$

$$\begin{aligned} 0 &\leq \rho \leq 2 \\ \rho &\leq u \leq 6 - \rho^2 \\ 0 &\leq \varphi \leq 2\pi \end{aligned}$$

$$\begin{aligned} V(\Omega) &= \iiint_{\Omega} dx dy dz = \iiint_{\Omega^*} \rho \cdot d\rho d\varphi du = \int_0^2 d\rho \int_{\rho}^{6-\rho^2} \rho \cdot du \cdot \int_0^{2\pi} d\varphi = \\ &= \int_0^2 [\rho \cdot u]_{\rho}^{6-\rho^2} \cdot d\rho \cdot [\varphi]_0^{2\pi} = 2\pi \cdot \int_0^2 (6\rho - \rho^3 - \rho^2) \cdot d\rho = \\ &= 2\pi \cdot \left[3\rho^2 - \frac{\rho^4}{4} - \frac{\rho^3}{3} \right]_0^2 = 2\pi \cdot \left(12 - 4 - \frac{8}{3} \right) = \underline{\underline{\frac{32\pi}{3}}} \end{aligned}$$

③ $\Omega: x^2 + y^2 + z^2 \leq 16, z^2 \leq 3x^2 + 3y^2, z \geq 0, V(\Omega) = ?$

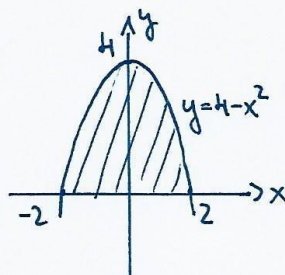
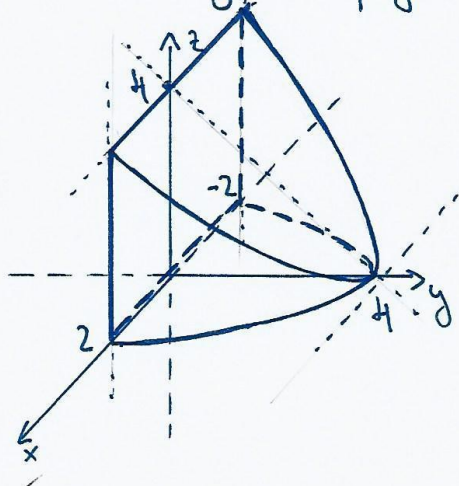


$$\begin{aligned}x &= \rho \cdot \cos\varphi \cdot \sin\psi \\y &= \rho \cdot \sin\varphi \cdot \sin\psi \\z &= \rho \cdot \cos\psi \\|\mathbf{v}| &= \rho^2 \cdot \sin\psi\end{aligned}$$

$$\begin{aligned}0 &\leq \rho \leq 4 \\0 &\leq \varphi \leq 2\pi \\\frac{\pi}{6} &\leq \psi \leq \frac{\pi}{2}\end{aligned}$$

$$\begin{aligned}V(\Omega) &= \iiint_{\Omega} dx dy dz = \iiint_{\Omega^*} \rho^2 \cdot \sin\psi \cdot d\rho d\varphi d\psi = \\&= \int_0^4 \rho^2 d\rho \cdot \int_0^{2\pi} d\varphi \cdot \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin\psi \cdot d\psi = \left[\frac{\rho^3}{3} \right]_0^4 \cdot \left[\varphi \right]_0^{2\pi} \cdot \left[-\cos\psi \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \\&= \frac{64}{3} \cdot 2\pi \cdot \frac{\sqrt{3}}{2} = \underline{\underline{\frac{64 \cdot \sqrt{3} \cdot \pi}{3}}}\end{aligned}$$

4) $\Omega: y=4-x^2, y+z=4, z=0, y=0, \sigma(x,y,z)=1, S_{xz}=?$



$$\begin{cases} -2 \leq x \leq 2 \\ 0 \leq y \leq 4-x^2 \\ 0 \leq z \leq 4-y \end{cases}$$

$$S_{xz} = \iiint_{\Omega} y \cdot \sigma(x,y,z) \cdot dx dy dz = \iiint_{\Omega} y \cdot dx dy dz =$$

$$= \int_{-2}^2 dx \int_0^{4-x^2} dy \int_0^{4-y} y \cdot dz = \int_{-2}^2 dx \int_0^{4-x^2} [yz]_0^{4-y} \cdot dy = \int_{-2}^2 dx \int_0^{4-x^2} (4y - y^2) dy =$$

$$= \int_{-2}^2 \left[2y^2 - \frac{y^3}{3} \right]_0^{4-x^2} \cdot dx = \int_{-2}^2 \left(2 \cdot (4-x^2)^2 - \frac{(4-x^2)^3}{3} \right) \cdot dx =$$

$$= \int_{-2}^2 \left(32 - 16x^2 + 2x^4 - \frac{64 - 48x^2 + 12x^4 - x^6}{3} \right) dx =$$

$$= \int_{-2}^2 \left(\frac{32}{3} - 2x^2 + \frac{x^6}{3} \right) dx = \left[\frac{32x}{3} - \frac{2x^3}{3} + \frac{x^7}{21} \right]_{-2}^2 =$$

$$= \frac{64}{3} - \frac{64}{3} + \frac{128}{21} + \frac{64}{3} - \frac{64}{3} + \frac{128}{21} = \underline{\underline{\frac{1024}{35}}}$$

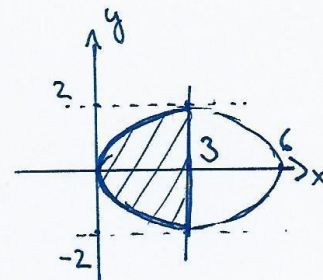
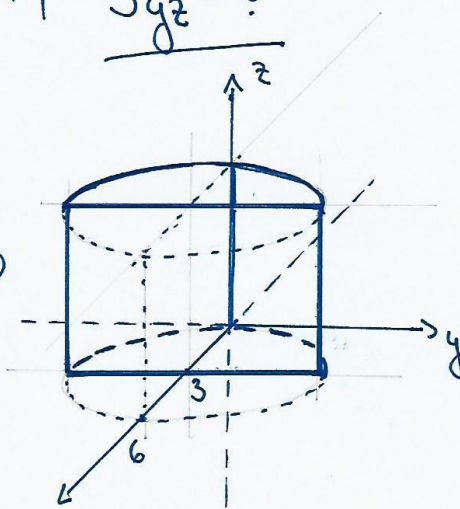
⑤ $\Omega: 4x^2 + 9y^2 \leq 24x, x \leq 3, z \geq 0, z \leq 4$
 $\sigma(x, y, z) = 1, S_{yz} = ?$

$$4x^2 + 9y^2 = 24x$$

$$4x^2 - 24x + 9y^2 = 0$$

$$4(x-3)^2 - 36 + 9y^2 = 0$$

$$\frac{(x-3)^2}{9} + \frac{y^2}{4} = 1$$



$$x = 3 \cdot \rho \cdot \cos \varphi + 3$$

$$y = 2 \cdot \rho \cdot \sin \varphi$$

$$z = u$$

$$J = 6\rho$$

$$\begin{array}{l} 0 \leq \rho \leq 1 \\ \frac{\pi}{2} \leq \varphi \leq \frac{3\pi}{2} \\ 0 \leq u \leq 4 \end{array}$$

$$S_{yz} = \iiint_{\Omega} x \cdot \sigma(x, y, z) \cdot dx dy dz =$$

$$= \iiint_{\Omega} x \cdot dx dy dz = \iiint_{\Omega^*} (3\rho \cos \varphi + 3) \cdot 6\rho \cdot d\rho d\varphi du =$$

$$= 18 \cdot \int_0^1 d\rho \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\varphi \int_0^4 (\rho^2 \cos \varphi + \rho) \cdot du = 18 \cdot \int_0^1 d\rho \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\rho^2 \cos \varphi + \rho) \cdot [u]_0^4 d\varphi$$

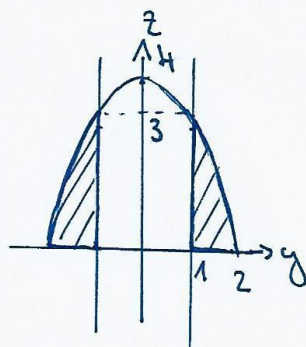
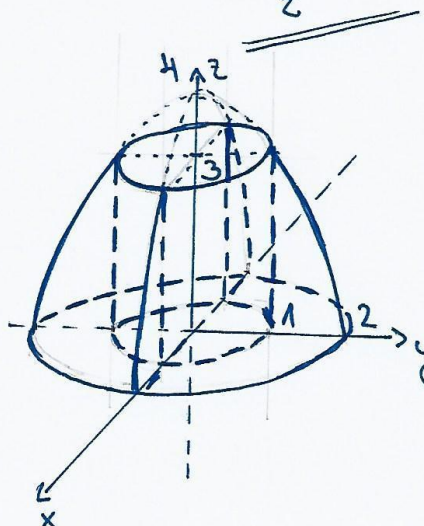
$$= 18 \cdot 4 \cdot \int_0^1 d\rho \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\rho^2 \cos \varphi + \rho) \cdot d\varphi = 72 \cdot \int_0^1 [\rho^2 \sin \varphi + \rho \varphi]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\rho =$$

$$= 72 \cdot \int_0^1 \left(-\rho^2 + \frac{3\pi}{2} \cdot \rho - \rho^2 - \frac{\pi}{2} \cdot \rho \right) d\rho = 72 \cdot \int_0^1 (\pi \cdot \rho - 2\rho^2) \cdot d\rho =$$

$$= 72 \cdot \left[\frac{\pi \rho^2}{2} - \frac{2\rho^3}{3} \right]_0^1 = 72 \left(\frac{\pi}{2} - \frac{2}{3} \right) = \underline{\underline{36\pi - 48}}$$

⑥ $\Omega: z \leq 4 - x^2 - y^2, x^2 + y^2 \geq 1, z \geq 0, \mathcal{G}(x, y, z) = x^2 + y^2$

$I_z = ?$



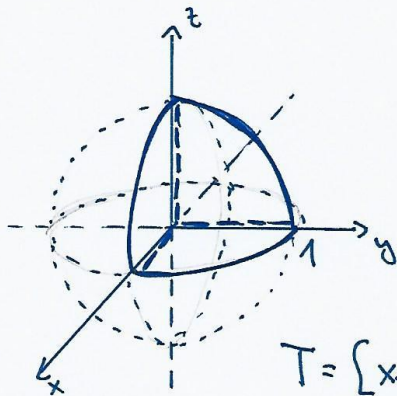
$$\begin{aligned} x &= \rho \cdot \cos \varphi \\ y &= \rho \cdot \sin \varphi \\ z &= u \\ \lambda &= \rho \end{aligned}$$

$$\begin{aligned} z &= 4 - x^2 - y^2 \\ u &= 4 - \rho^2 \cos^2 \varphi - \rho^2 \sin^2 \varphi \\ u &= 4 - \rho^2 \end{aligned}$$

$$\begin{aligned} 1 &\leq \rho \leq 2 \\ 0 &\leq u \leq 4 - \rho^2 \\ 0 &\leq \varphi \leq 2\pi \end{aligned}$$

$$\begin{aligned} I_z &= \iiint_{\Omega} (x^2 + y^2) \cdot \mathcal{G}(x, y, z) \cdot dx dy dz = \iiint_{\Omega} (x^2 + y^2)^2 \cdot dx dy dz = \\ &= \iiint_{\Omega^*} (\rho^2 \cos^2 \varphi + \rho^2 \sin^2 \varphi)^2 \cdot \rho \cdot d\rho d\varphi du = \int_1^2 d\rho \int_0^{4-\rho^2} \rho^5 \cdot du \cdot \int_0^{2\pi} d\varphi = \\ &= \int_1^2 [\rho^5 \cdot u]_0^{4-\rho^2} \cdot d\rho \cdot [\varphi]_0^{2\pi} = 2\pi \cdot \int_1^2 (4\rho^5 - \rho^7) \cdot d\rho = \\ &= 2\pi \cdot \left[\frac{2\rho^6}{3} - \frac{\rho^8}{8} \right]_1^2 = 2\pi \cdot \left(\frac{128}{3} - 32 - \frac{2}{3} + \frac{1}{8} \right) = \\ &= \frac{81\pi}{4} \end{aligned}$$

④ $\Omega: x^2 + y^2 + z^2 \leq 1, x \geq 0, y \geq 0, z \geq 0, f(x, y, z) = 1 \quad T = ?$



$$\begin{aligned} x &= \rho \cdot \cos \varphi \cdot \sin \psi \\ y &= \rho \cdot \sin \varphi \cdot \sin \psi \\ z &= \rho \cdot \cos \psi \\ |\mathbf{s}| &= \rho^2 \cdot \sin \psi \end{aligned}$$

$$\begin{aligned} 0 &\leq \rho \leq 1 \\ 0 &\leq \varphi \leq \frac{\pi}{2} \\ 0 &\leq \psi \leq \frac{\pi}{2} \end{aligned}$$

$$T = [x_T, y_T, z_T] \quad x_T = y_T = z_T$$

$$\begin{aligned} m &= \iiint_{\Omega} f(x, y, z) \cdot dx dy dz = \iiint_{\Omega} dx dy dz = \iiint_{\Omega^*} \rho^2 \cdot \sin \psi \cdot d\rho d\varphi d\psi = \\ &= \int_0^1 \rho^2 \cdot d\rho \cdot \int_0^{\frac{\pi}{2}} d\varphi \cdot \int_0^{\frac{\pi}{2}} \sin \psi \cdot d\psi = \left[\frac{\rho^3}{3} \right]_0^1 \cdot [\varphi]_0^{\frac{\pi}{2}} \cdot [-\cos \psi]_0^{\frac{\pi}{2}} = \\ &= \frac{1}{3} \cdot \frac{\pi}{2} \cdot 1 = \frac{\pi}{6} \end{aligned}$$

$$\begin{aligned} S_{xy} &= \iiint_{\Omega} z \cdot f(x, y, z) \cdot dx dy dz = \iiint_{\Omega} z \cdot dx dy dz = \\ &= \iiint_{\Omega^*} \rho \cdot \cos \psi \cdot \rho^2 \cdot \sin \psi \cdot d\rho d\varphi d\psi = \int_0^1 \rho^3 \cdot d\rho \cdot \int_0^{\frac{\pi}{2}} d\varphi \cdot \int_0^{\frac{\pi}{2}} \cos \psi \cdot \sin \psi \cdot d\psi = \\ &= \left[\frac{\rho^4}{4} \right]_0^1 \cdot [\varphi]_0^{\frac{\pi}{2}} \cdot \int_0^{\frac{\pi}{2}} \frac{\sin 2\psi}{2} \cdot d\psi = \frac{1}{4} \cdot \frac{\pi}{2} \cdot \left[-\frac{\cos 2\psi}{4} \right]_0^{\frac{\pi}{2}} = \\ &= \frac{\pi}{8} \cdot \left(\frac{1}{4} + \frac{1}{4} \right) = \frac{\pi}{16} \end{aligned}$$

$$z_T = \frac{S_{xy}}{m} = \frac{\frac{\pi}{16}}{\frac{\pi}{6}} = \frac{3}{8}$$

$$\underline{\underline{T = \left[\frac{3}{8} \mid \frac{3}{8} \mid \frac{3}{8} \right]}}$$