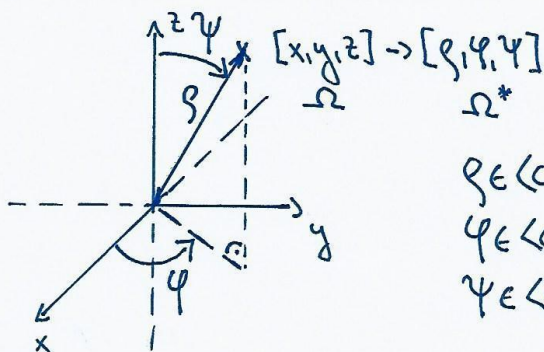


$$\begin{aligned}x &= \rho \cdot \cos\varphi \cdot \sin\psi \\y &= \rho \cdot \sin\varphi \cdot \sin\psi \\z &= \rho \cdot \cos\psi \\|J| &= \rho^2 \sin\psi\end{aligned}$$



$$x^2 + y^2 + z^2 = \rho^2$$

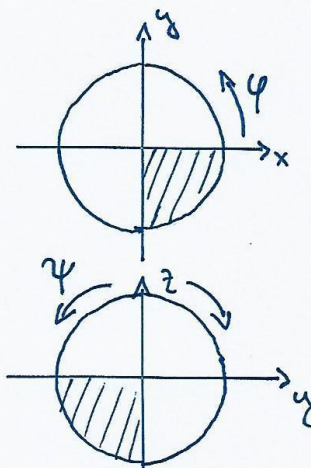
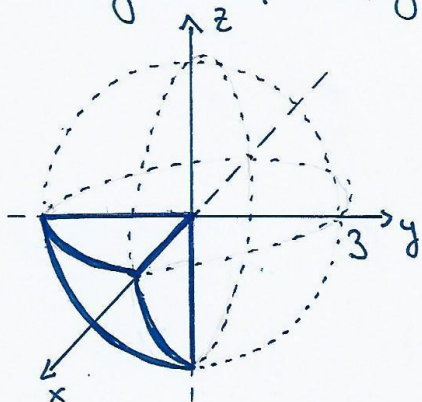
$$\rho \in (0, \infty)$$

$$\varphi \in (0, 2\pi)$$

$$\psi \in (0, \pi) \quad \nabla$$

$$\textcircled{1} \iiint_{\Omega} x \cdot dx dy dz = \iiint_{\Omega^*} \rho \cdot \cos\varphi \cdot \sin\psi \cdot \rho^2 \sin\psi \cdot d\rho d\varphi d\psi =$$

$$\Omega: x^2 + y^2 + z^2 \leq 9, x \geq 0, y \leq 0, z \leq 0$$



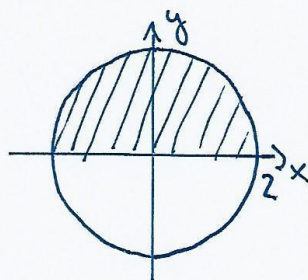
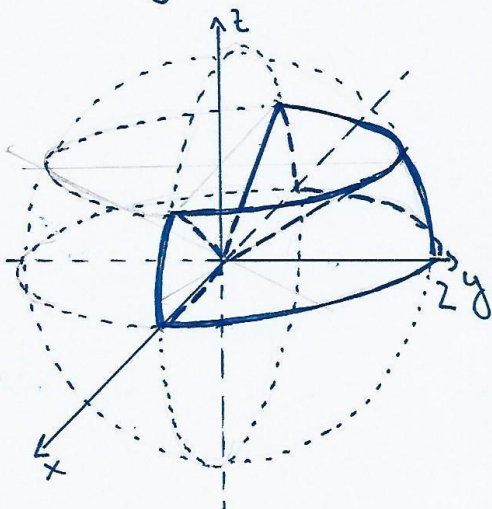
$$\begin{aligned}x &= \rho \cdot \cos\varphi \cdot \sin\psi \\y &= \rho \cdot \sin\varphi \cdot \sin\psi \\z &= \rho \cdot \cos\psi \\|J| &= \rho^2 \sin\psi\end{aligned}$$

$$\begin{aligned}0 &\leq \rho \leq 3 \\ \frac{3\pi}{2} &\leq \varphi \leq 2\pi \\ \frac{\pi}{2} &\leq \psi \leq \pi\end{aligned}$$

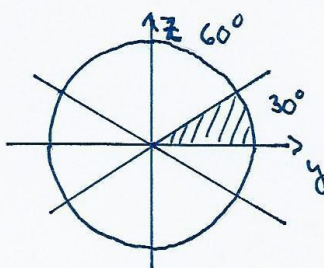
$$\begin{aligned}&= \int_0^3 \rho^3 d\rho \cdot \int_{\frac{3\pi}{2}}^{2\pi} \cos\varphi d\varphi \cdot \int_{\frac{\pi}{2}}^{\pi} \sin^2\psi d\psi = \left[\frac{\rho^4}{4} \right]_0^3 \cdot \left[\sin\varphi \right]_{\frac{3\pi}{2}}^{2\pi} \cdot \int_{\frac{\pi}{2}}^{\pi} \frac{1 - \cos 2\psi}{2} d\psi = \\&= \frac{81}{4} \cdot (0 + 1) \cdot \frac{1}{2} \cdot \left[\psi - \frac{\sin 2\psi}{2} \right]_{\frac{\pi}{2}}^{\pi} = \frac{81}{8} \cdot \left(\pi - 0 - \frac{\pi}{2} + 0 \right) = \\&= \underline{\underline{\frac{81\pi}{16}}}\end{aligned}$$

$$(2) \iiint_{\Omega} (x^2 + y^2 + z^2) \cdot dx dy dz =$$

$$\Omega: x^2 + y^2 + z^2 \leq 4, x^2 + y^2 \geq 3z^2, y \geq 0, z \geq 0$$



$$\begin{aligned} x &= \rho \cdot \cos\varphi \cdot \sin\psi \\ y &= \rho \cdot \sin\varphi \cdot \sin\psi \\ z &= \rho \cdot \cos\psi \\ |\mathbf{s}| &= \rho^2 \cdot \sin\psi \end{aligned}$$



$$\begin{aligned} 0 &\leq \rho \leq 2 \\ 0 &\leq \varphi \leq \pi \\ \frac{\pi}{3} &\leq \psi \leq \frac{\pi}{2} \end{aligned}$$

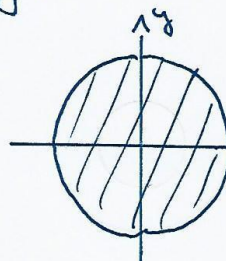
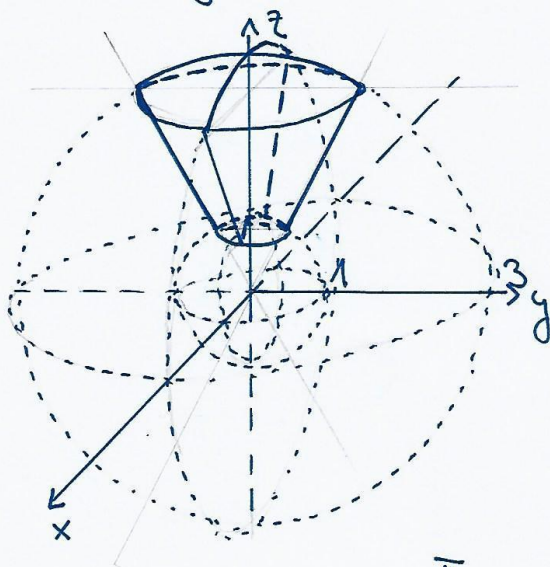
$$= \iiint_{\Omega^*} \underbrace{(\rho^2 \cos^2\varphi \sin^2\psi + \rho^2 \sin^2\varphi \sin^2\psi + \rho^2 \cos^2\psi)}_{\rho^2 \sin^2\psi} \cdot \rho^2 \sin\psi \cdot d\rho d\varphi d\psi =$$

$$= \int_0^2 \rho^4 \cdot d\rho \cdot \int_0^\pi d\varphi \cdot \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin\psi \cdot d\psi = \left[\frac{\rho^5}{5} \right]_0^2 \cdot [\varphi]_0^\pi \cdot [-\cos\psi]_{\frac{\pi}{3}}^{\frac{\pi}{2}} =$$

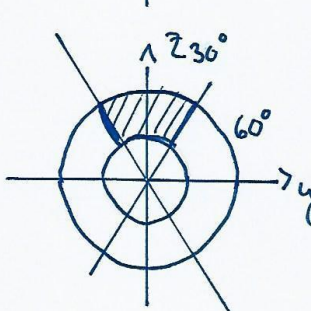
$$= \frac{32}{5} \cdot \pi \cdot \frac{1}{2} = \underline{\underline{\frac{16\pi}{5}}}$$

$$\textcircled{3} \iiint_{\Omega} (x^2 + y^2) \cdot dx dy dz = \iiint_{\Omega^*} (\rho^2 \cos^2 \varphi \cdot \sin^2 \psi + \rho^2 \sin^2 \varphi \cdot \sin^2 \psi) \cdot \rho^2 \sin \psi \cdot d\rho d\varphi d\psi =$$

$$\Omega: 1 \leq x^2 + y^2 + z^2 \leq 9, z^2 \geq 3x^2 + 3y^2, z \geq 0$$



$$\begin{aligned} x &= \rho \cdot \cos \varphi \cdot \sin \psi \\ y &= \rho \cdot \sin \varphi \cdot \sin \psi \\ z &= \rho \cdot \cos \psi \\ |\mathbf{s}| &= \rho^2 \cdot \sin \psi \end{aligned}$$



$$\begin{aligned} 1 &\leq \rho \leq 3 \\ 0 &\leq \varphi \leq 2\pi \\ 0 &\leq \psi \leq \frac{\pi}{6} \end{aligned}$$

$$= \int_1^3 \rho^4 \cdot d\rho \cdot \int_0^{2\pi} d\varphi \cdot \int_0^{\frac{\pi}{6}} \sin^3 \psi \cdot d\psi =$$

$$= \left[\frac{\rho^5}{5} \right]_1^3 \cdot \left[\varphi \right]_0^{2\pi} \cdot \int_0^{\frac{\pi}{6}} (1 - \cos^2 \psi) \cdot \sin \psi \cdot d\psi = \left| \begin{array}{l} \cos \psi = k \\ -\sin \psi \cdot d\psi = dk \\ \psi = \frac{\pi}{6} \Rightarrow k = \frac{\sqrt{3}}{2} \\ \psi = 0 \Rightarrow k = 1 \end{array} \right| =$$

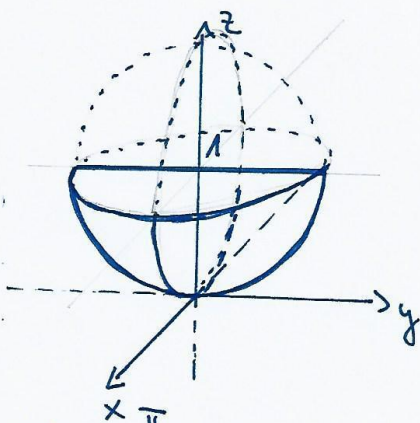
$$\frac{1}{5} \cdot (243 - 1) \cdot 2\pi \cdot \int_1^{\frac{\sqrt{3}}{2}} (1 - k^2) \cdot (-dk) = \frac{484\pi}{5} \cdot \left[-k + \frac{k^3}{3} \right]_1^{\frac{\sqrt{3}}{2}} =$$

$$= \frac{484\pi}{5} \cdot \left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{8} + 1 - \frac{1}{3} \right) = \underline{\underline{\frac{484\pi}{5} \left(\frac{2}{3} - \frac{3\sqrt{3}}{8} \right)}}$$

$$\textcircled{4} \quad \iiint_{\Omega} z \cdot dx dy dz = \iiint_{\Omega^*} (\rho \cdot \cos \varphi + 1) \cdot \rho^2 \cdot \sin \varphi \cdot d\rho d\varphi d\psi =$$

$$\Omega: x^2 + y^2 + z^2 \leq 2z, z \leq 1, x \geq 0$$

$$(x^2 + y^2 + (z-1)^2 \leq 1)$$



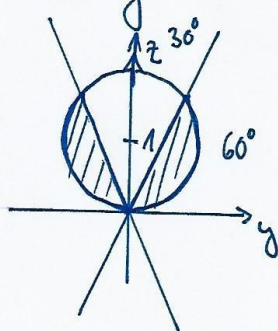
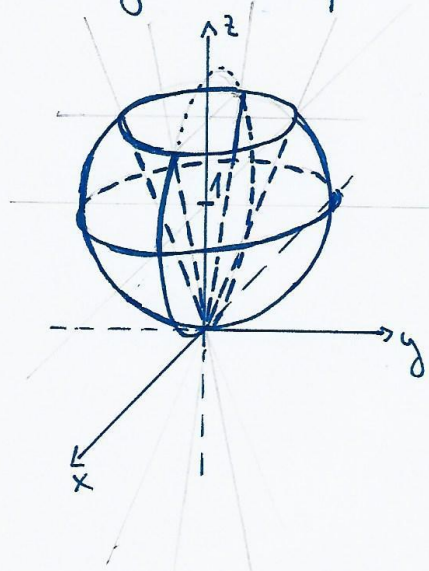
$$\begin{aligned} x &= \rho \cdot \cos \varphi \cdot \sin \psi \\ y &= \rho \cdot \sin \varphi \cdot \sin \psi \\ z &= \rho \cdot \cos \psi + 1 \\ |d\mathbf{r}| &= \rho^2 \cdot \sin \psi \end{aligned}$$

$$\begin{aligned} 0 &\leq \rho \leq 1 \\ -\frac{\pi}{2} &\leq \varphi \leq \frac{\pi}{2} \\ \frac{\pi}{2} &\leq \psi \leq \pi \end{aligned}$$

$$\begin{aligned} &\int_0^1 d\rho \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\rho^3 \cos \psi \cdot \sin \psi + \rho^2 \cdot \sin \psi) \cdot d\varphi \cdot \int_{\frac{\pi}{2}}^{\pi} d\psi = \\ &= \int_0^1 d\rho \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\rho^3 \cdot \frac{\sin 2\psi}{2} + \rho^2 \cdot \sin \psi \right) d\varphi \cdot [\varphi]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \\ &= \pi \cdot \int_0^1 \left[-\frac{\cos 2\psi}{4} \cdot \rho^3 - \rho^2 \cos \psi \right]_{\frac{\pi}{2}}^{\pi} d\rho = \\ &= \pi \cdot \int_0^1 \left(-\frac{\rho^3}{4} + \rho^2 - \frac{\rho^3}{4} + 0 \right) d\rho = \pi \cdot \int_0^1 \left(\rho^2 - \frac{\rho^3}{2} \right) d\rho = \\ &= \pi \cdot \left[\frac{\rho^3}{3} - \frac{\rho^4}{8} \right]_0^1 = \pi \cdot \left(\frac{1}{3} - \frac{1}{8} \right) = \underline{\underline{\frac{5\pi}{24}}} \end{aligned}$$

$$\textcircled{5} \quad \iiint_{\Omega} dx dy dz = \iiint_{\Omega^*} \rho^2 \cdot \sin \psi \cdot d\rho d\psi d\varphi =$$

$$\Omega: x^2 + y^2 + z^2 \leq 2z, \quad z^2 \leq 3x^2 + 3y^2$$



$$x = \rho \cdot \cos \varphi \cdot \sin \psi$$

$$y = \rho \cdot \sin \varphi \cdot \sin \psi$$

$$z = \rho \cdot \cos \psi$$

$$|\mathbf{J}| = \rho^2 \sin \psi$$

$$x^2 + y^2 + z^2 = 2z$$

$$\rho^2 = 2\rho \cdot \cos \psi$$

$$\rho = 2 \cos \psi$$

$$\frac{\pi}{6} \leq \psi \leq \frac{\pi}{2}$$

$$0 \leq \rho \leq 2 \cos \psi$$

$$0 \leq \varphi \leq 2\pi$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} d\psi \int_0^{2 \cos \psi} \rho^2 \sin \psi \cdot d\rho \cdot \int_0^{2\pi} d\varphi = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left[\frac{\rho^3}{3} \cdot \sin \psi \right]_0^{2 \cos \psi} \cdot d\psi \cdot [\varphi]_0^{2\pi} =$$

$$= 2\pi \cdot \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{8}{3} \cdot \cos^3 \psi \cdot \sin \psi \cdot d\psi = \left. \begin{array}{l} \cos \psi = k \\ -\sin \psi \cdot d\psi = dk \\ \psi = \frac{\pi}{2} \Rightarrow k = 0 \\ \psi = \frac{\pi}{6} \Rightarrow k = \frac{\sqrt{3}}{2} \end{array} \right| =$$

$$= 2\pi \cdot \frac{8}{3} \cdot \int_{\frac{\sqrt{3}}{2}}^0 k^3 \cdot (-dk) = \frac{16\pi}{3} \cdot \left[\frac{k^4}{4} \right]_0^{\frac{\sqrt{3}}{2}} = \frac{16\pi}{3} \cdot \frac{9}{4 \cdot 16} =$$

$$= \underline{\underline{\frac{3\pi}{4}}}$$