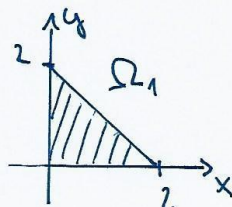
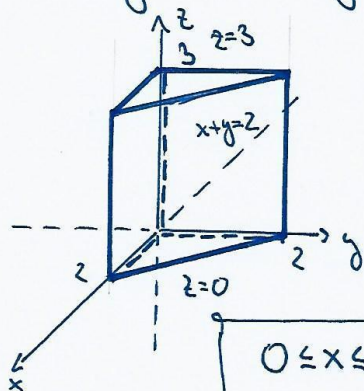


$$\textcircled{1} \iiint_{\Omega} y \cdot dx dy dz = \int_0^2 dx \int_0^{2-x} dy \int_0^3 y \cdot dz = \int_0^2 dx \int_0^{2-x} [yz]_0^3 \cdot dy =$$

$$\Omega: x+y=2, x=0, y=0, z=0, z=3 \quad = \int_0^2 dx \int_0^{2-x} 3y \cdot dy =$$



$$\begin{aligned} 0 &\leq x \leq 2 \\ 0 &\leq y \leq 2-x \\ 0 &\leq z \leq 3 \end{aligned}$$

$$= 3 \cdot \int_0^2 \left[\frac{y^2}{2} \right]_0^{2-x} \cdot dx =$$

$$= 3 \cdot \int_0^2 \frac{(2-x)^2}{2} \cdot dx =$$

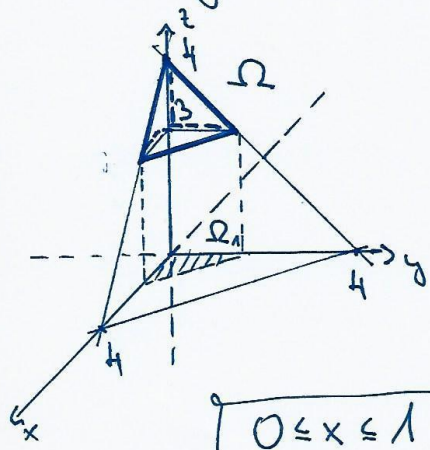
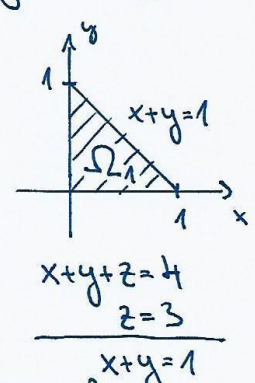
$$= 3 \cdot \int_0^2 \left(2 - 2x + \frac{x^2}{2} \right) \cdot dx =$$

$$= 3 \cdot \left[2x - x^2 + \frac{x^3}{6} \right]_0^2 =$$

$$= 3 \cdot \left(4 - 4 + \frac{8}{6} \right) = \underline{\underline{4}}$$

(2)
$$\iiint_{\Omega} x \cdot dx dy dz = \int_0^1 dx \int_0^{1-x} dy \int_3^{4-x-y} x \cdot dz = \int_0^1 dx \int_0^{1-x} [x \cdot z]_3^{4-x-y} dy =$$

$\Omega: x+y+z=4, x=0, y=0, z=3$

$$= \int_0^1 dx \int_0^{1-x} (x \cdot (4-x-y) - 3x) dy =$$

$$= \int_0^1 dx \int_0^{1-x} (x - x^2 - xy) dy =$$

$$= \int_0^1 \left[xy - x^2 y - \frac{xy^2}{2} \right]_0^{1-x} dx =$$

$$= \int_0^1 \left(x(1-x) - x^2(1-x) - \frac{x(1-x)^2}{2} \right) dx =$$

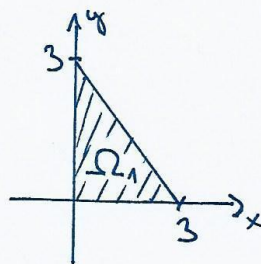
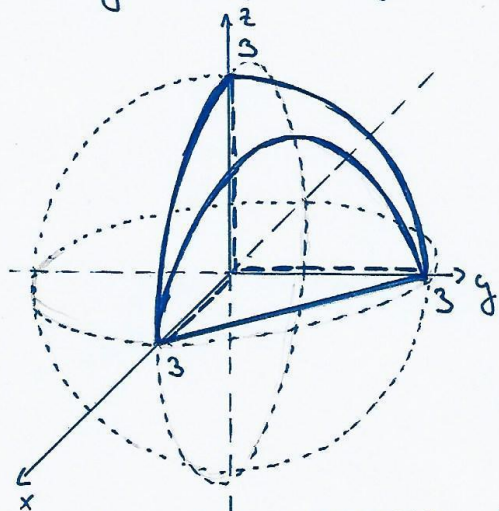
$$= \int_0^1 \left(x - x^2 - x^2 + x^3 - \frac{x}{2} + x^2 - \frac{x^3}{2} \right) dx = \int_0^1 \left(\frac{x^3}{2} - x^2 + \frac{x}{2} \right) dx =$$

$$= \left[\frac{x^4}{8} - \frac{x^3}{3} + \frac{x^2}{4} \right]_0^1 = \frac{1}{8} - \frac{1}{3} + \frac{1}{4} = \underline{\underline{\frac{1}{24}}}$$

$$\begin{aligned} 0 &\leq x \leq 1 \\ 0 &\leq y \leq 1-x \\ 3 &\leq z \leq 4-x-y \end{aligned}$$

$$\textcircled{3} \quad \iiint_{\Omega} 2z \cdot dx dy dz = \int_0^3 dx \int_0^{3-x} dy \int_0^{\sqrt{9-x^2-y^2}} 2z \cdot dz = (*)$$

$$\Omega: x^2 + y^2 + z^2 \leq 9, x \geq 0, y \geq 0, z \geq 0, x+y \leq 3$$



$$\begin{array}{l} 0 \leq x \leq 3 \\ 0 \leq y \leq 3-x \\ 0 \leq z \leq \sqrt{9-x^2-y^2} \end{array}$$

$$(*) = \int_0^3 dx \int_0^{3-x} \left[z^2 \right]_0^{\sqrt{9-x^2-y^2}} \cdot dy = \int_0^3 dx \int_0^{3-x} (9-x^2-y^2) \cdot dy =$$

$$= \int_0^3 \left[9y - x^2y - \frac{y^3}{3} \right]_0^{3-x} dx = \int_0^3 \left(9(3-x) - x^2(3-x) - \frac{(3-x)^3}{3} \right) dx =$$

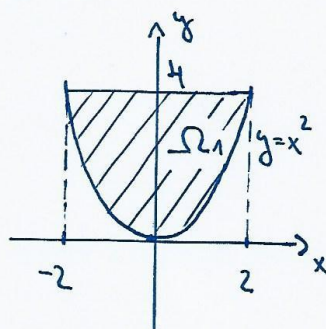
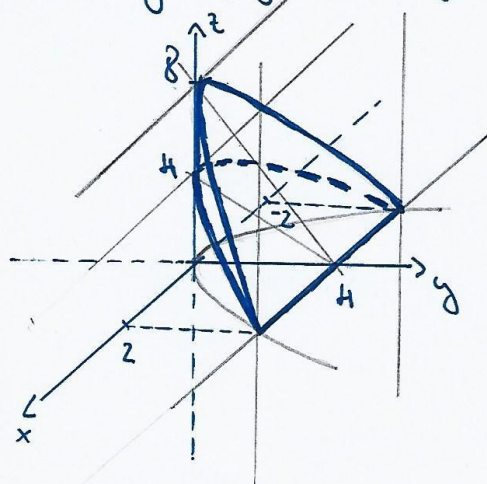
$$= \int_0^3 \left(27 - 9x - 3x^2 + x^3 - 9 + 9x - 3x^2 + \frac{x^3}{3} \right) dx =$$

$$= \int_0^3 \left(18 - 6x^2 + \frac{4x^3}{3} \right) dx = \left[18x - 2x^3 + \frac{x^4}{3} \right]_0^3 =$$

$$= 54 - 54 + 27 = \underline{\underline{27}}$$

$$\textcircled{4} \quad \iiint_{\Omega} x^2 \cdot dx dy dz = \int_{-2}^2 dx \int_{x^2}^4 dy \int_{4-y}^{8-2y} x^2 \cdot dz = \int_{-2}^2 dx \int_{x^2}^4 [x^2 z]_{4-y}^{8-2y} \cdot dy =$$

$$\Omega: y=x^2, y+z=4, 2y+z=8$$



$$\begin{aligned} -2 \leq x \leq 2 \\ x^2 \leq y \leq 4 \\ 4-y \leq z \leq 8-2y \end{aligned}$$

$$\begin{aligned} &= \int_{-2}^2 dx \int_{x^2}^4 (x^2(8-2y) - x^2(4-y)) dy = \int_{-2}^2 dx \int_{x^2}^4 (4x^2 - x^2 y) dy = \\ &= \int_{-2}^2 \left[4x^2 y - \frac{x^2 y^2}{2} \right]_{x^2}^4 dx = \int_{-2}^2 \left(16x^2 - 8x^2 - 4x^4 + \frac{x^6}{2} \right) dx = \end{aligned}$$

$$= \int_{-2}^2 \left(8x^2 - 4x^4 + \frac{x^6}{2} \right) dx = \left[\frac{8x^3}{3} - \frac{4x^5}{5} + \frac{x^7}{14} \right]_{-2}^2 =$$

$$= 2 \left(\frac{64}{3} - \frac{128}{5} + \frac{64}{7} \right) = 2 \cdot \frac{512}{105} = \frac{1024}{105}$$