

$$x^2 + y^2 = r^2$$

$$r = \sqrt{x^2 + y^2}$$

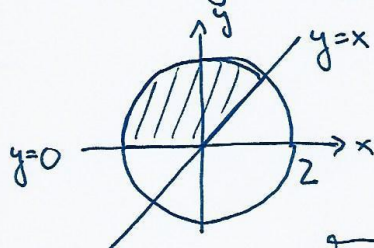
$$\frac{y}{x} = \tan \varphi \quad \varphi = \arctan\left(\frac{y}{x}\right) \quad x \neq 0$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} \end{vmatrix} = \begin{vmatrix} \cos \varphi & -r \sin \varphi \\ \sin \varphi & r \cos \varphi \end{vmatrix} = r \cdot \cos^2 \varphi + r \cdot \sin^2 \varphi = r$$

$$\underline{dx dy = r \cdot dr d\varphi}$$

$$\textcircled{1} \iint_{\Omega} (x+y) dx dy = \iint_{\Omega^*} (r \cdot \cos \varphi + r \cdot \sin \varphi) r \cdot dr d\varphi =$$

$$\Omega: x^2 + y^2 \leq 4, y \geq x, y \geq 0 = \iint_{\Omega^*} r^2 \cdot (\cos \varphi + \sin \varphi) \cdot dr d\varphi =$$



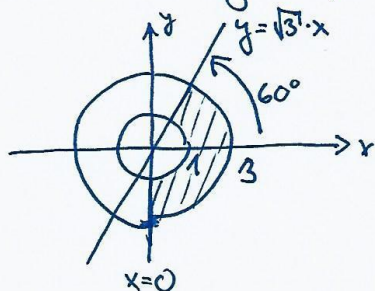
$$\begin{aligned} x &= r \cdot \cos \varphi \\ y &= r \cdot \sin \varphi \\ J &= r \end{aligned}$$

$$\begin{cases} 0 \leq r \leq 2 \\ \frac{\pi}{4} \leq \varphi \leq \pi \end{cases} \Omega^*$$

$$\begin{aligned} &= \int_0^2 r^2 \cdot dr \cdot \int_{\frac{\pi}{4}}^{\pi} (\cos \varphi + \sin \varphi) \cdot d\varphi = \\ &= \left[\frac{r^3}{3} \right]_0^2 \cdot \left[\sin \varphi - \cos \varphi \right]_{\frac{\pi}{4}}^{\pi} = \\ &= \frac{8}{3} \cdot \left(0 + 1 - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) = \underline{\underline{\frac{8}{3}}} \end{aligned}$$

$$\textcircled{2} \iint_{\Omega} x \cdot dx dy = \iint_{\Omega^*} \rho \cdot \cos \varphi \cdot \rho \cdot d\rho d\varphi = \iint_{\Omega^*} \rho^2 \cdot \cos \varphi \cdot d\rho d\varphi =$$

$$\Omega: 1 \leq x^2 + y^2 \leq 9, x \geq 0, y \leq \sqrt{3} \cdot x$$



$$y = \sqrt{3} \cdot x \Rightarrow \operatorname{tg} \varphi = \sqrt{3} \Rightarrow \varphi = \frac{\pi}{3}$$

$$x = \rho \cdot \cos \varphi$$

$$y = \rho \cdot \sin \varphi$$

$$\rho = \rho$$

$$\boxed{1 \leq \rho \leq 3}$$

$$\boxed{-\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{3}}$$

$$= \int_1^3 \rho^2 \cdot d\rho \cdot \int_{-\frac{\pi}{2}}^{\frac{\pi}{3}} \cos \varphi \cdot d\varphi =$$

$$= \left[\frac{\rho^3}{3} \right]_1^3 \cdot \left[\sin \varphi \right]_{-\frac{\pi}{2}}^{\frac{\pi}{3}} =$$

$$= \left(\frac{27}{3} - \frac{1}{3} \right) \cdot \left(\frac{\sqrt{3}}{2} + 1 \right) =$$

$$= \frac{26}{3} \cdot \frac{\sqrt{3}+2}{2} = \frac{13(\sqrt{3}+2)}{3}$$

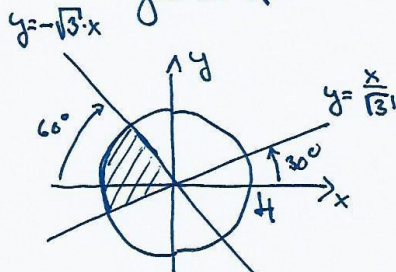
$$\boxed{\begin{aligned} \operatorname{tg} \varphi = \sqrt{3} &\Rightarrow \varphi = \frac{\pi}{3} \\ \operatorname{tg} \varphi = 1 &\Rightarrow \varphi = \frac{\pi}{4} \\ \operatorname{tg} \varphi = \frac{1}{\sqrt{3}} &\Rightarrow \varphi = \frac{\pi}{6} \end{aligned}}$$

$$\textcircled{3} \iint_{\Omega} (x^2 + y^2) dx dy = \iint_{\Omega^*} (\rho^2 \cos^2 \varphi + \rho^2 \sin^2 \varphi) \cdot \rho \cdot d\rho d\varphi =$$

$$\Omega: x^2 + y^2 \leq 16$$

$$y \geq \frac{x}{\sqrt{3}}$$

$$y \leq \sqrt{3} \cdot x$$



$$x = \rho \cdot \cos \varphi$$

$$y = \rho \cdot \sin \varphi$$

$$\rho = \rho$$

$$\boxed{0 \leq \rho \leq 4}$$

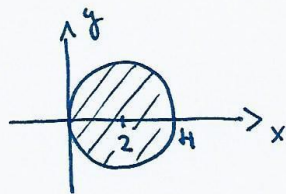
$$\boxed{\frac{2\pi}{3} \leq \varphi \leq \frac{4\pi}{3}}$$

$$= \iint_{\Omega^*} \rho^3 \cdot d\rho d\varphi = \int_0^4 \rho^3 \cdot d\rho \cdot \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} d\varphi =$$

$$= \left[\frac{\rho^4}{4} \right]_0^4 \cdot \left[\varphi \right]_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} =$$

$$= 64 \cdot \left(\frac{4\pi}{3} - \frac{2\pi}{3} \right) = \underline{\underline{32\pi}}$$

④



$$\begin{aligned} a) \quad x &= \rho \cdot \cos \varphi \\ y &= \rho \cdot \sin \varphi \\ \rho &= \rho \end{aligned}$$

$$(x-2)^2 + y^2 = 4$$

$$x^2 - 4x + 4 + y^2 = 4$$

$$x^2 + y^2 = 4x$$

$$\rho^2 \cdot \cos^2 \varphi + \rho^2 \cdot \sin^2 \varphi = 4 \rho \cos \varphi$$

$$\rho^2 = 4 \rho \cos \varphi$$

$$\rho = 4 \cos \varphi$$

$$\begin{aligned} -\frac{\pi}{2} &\leq \varphi \leq \frac{\pi}{2} \\ 0 &\leq \rho \leq 4 \cos \varphi \end{aligned}$$

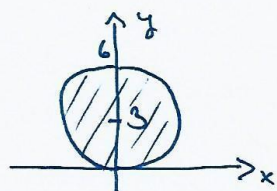
$$\begin{aligned} b) \quad x &= \rho \cdot \cos \varphi + 2 \\ y &= \rho \cdot \sin \varphi \\ \rho &= \rho \end{aligned}$$

$$\begin{aligned} 0 &\leq \rho \leq 2 \\ 0 &\leq \varphi \leq 2\pi \end{aligned}$$

$$x^2 + y^2 = \rho^2$$

$$\textcircled{5} \quad \iint_{\Omega} y \cdot dx dy = \iint_{\Omega^*} (\rho \cdot \sin \varphi + 3) \rho \cdot d\rho d\varphi =$$

$$\Omega: x^2 + y^2 \leq 6y$$



$$\begin{aligned} x &= \rho \cdot \cos \varphi \\ y &= \rho \cdot \sin \varphi + 3 \\ \rho &= \rho \end{aligned}$$

$$\begin{aligned} 0 &\leq \rho \leq 3 \\ 0 &\leq \varphi \leq 2\pi \end{aligned}$$

$$\begin{aligned} &= \int_0^3 d\rho \int_0^{2\pi} (\rho^2 \cdot \sin \varphi + 3\rho) \cdot d\varphi = \\ &= \int_0^3 \left[-\rho^2 \cdot \cos \varphi + 3\rho \varphi \right]_0^{2\pi} d\rho = \end{aligned}$$

$$= \int_0^3 (-\rho^2 + 6\pi \rho + \rho^2) d\rho =$$

$$= \int_0^3 6\pi \rho \cdot d\rho = \left[6\pi \cdot \frac{\rho^2}{2} \right]_0^3 =$$

$$= 6\pi \cdot \frac{9}{2} = \underline{\underline{27\pi}}$$

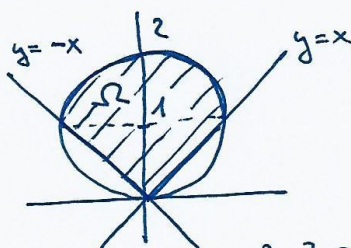
⑥ $\iint_{\Omega} dx dy = \iint_{\Omega^*} \rho \cdot d\rho d\varphi = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\varphi \int_0^{2\sin\varphi} \rho \cdot d\rho = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left[\frac{\rho^2}{2} \right]_0^{2\sin\varphi} d\varphi =$

$\Omega: x^2 + y^2 \leq 2y, y^2 \geq x^2$
 $(y^2 = x^2 \Leftrightarrow y = \pm x)$

$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 2\sin^2\varphi \cdot d\varphi = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (1 - \cos 2\varphi) d\varphi =$

$= \left[\varphi - \frac{\sin 2\varphi}{2} \right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \frac{3\pi}{4} - \frac{\sin \frac{3\pi}{2}}{2} - \frac{\pi}{4} + \frac{\sin \frac{\pi}{2}}{2} =$

$= \frac{3\pi}{4} + 1 - \frac{\pi}{4} + 1 = \underline{\underline{\frac{\pi}{2} + 2}}$



$x = \rho \cdot \cos\varphi$
 $y = \rho \cdot \sin\varphi$
 $\rho = \rho$

$x^2 + y^2 = 2y$
 $\rho^2 = 2\rho \sin\varphi$
 $\rho = 2\sin\varphi$

$\frac{\pi}{4} \leq \varphi \leq \frac{3\pi}{4}$
 $0 \leq \rho \leq 2\sin\varphi$

$\left(\begin{array}{l} \sin^2\varphi = \frac{1 - \cos 2\varphi}{2} \\ \cos^2\varphi = \frac{1 + \cos 2\varphi}{2} \end{array} \right)$

⑦ $\iint_{\Omega} x \cdot y \cdot dx dy = \iint_{\Omega^*} 3 \cdot \rho \cdot \cos\varphi \cdot 2 \cdot \rho \cdot \sin\varphi \cdot 6\rho \cdot d\rho d\varphi =$

$\Omega: 4x^2 + 9y^2 \leq 36, y \leq 0$
 $\left(\frac{x^2}{9} + \frac{y^2}{4} \leq 1 \right)$ elipse $a=3, b=2$

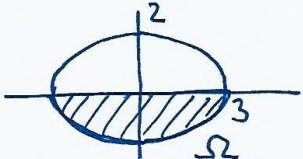
$= 36 \iint_{\Omega^*} \rho^3 \cdot \cos\varphi \cdot \sin\varphi \cdot d\rho d\varphi =$

$= 36 \cdot \int_0^1 \rho^3 \cdot d\rho \cdot \int_{\pi}^{2\pi} \frac{\sin 2\varphi}{2} \cdot d\varphi =$

$= 36 \cdot \left[\frac{\rho^4}{4} \right]_0^1 \cdot \left[-\frac{\cos 2\varphi}{4} \right]_{\pi}^{2\pi} =$

$= 36 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot (-\cos 4\pi + \cos 2\pi) =$

$= \frac{9}{4} \cdot (-1 + 1) = \underline{\underline{0}}$



$x = 3\rho \cdot \cos\varphi$
 $y = 2\rho \cdot \sin\varphi$
 $\rho = 3 \cdot 2 \cdot \rho = 6\rho$

$0 \leq \rho \leq 1$
 $\pi \leq \varphi \leq 2\pi$